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STRESS FUNCTIONS FOR A COSSERAT CONTINUUM

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Abstract—In this paper, a complete solution, in terms of stress functions analogous to the Papkovitch functions of classical elasticity, is obtained for the linear equations of an isotropic, elastic, Cosserat continuum. The special solutions for the concentrated force and couple are also given.

EQUATIONS OF A COSSERAT CONTINUUM

AT EACH point of a Cosserat continuum [1] there is a micro-structure which can rotate with respect to the surrounding medium. In the linear, elastic case, the potential energy-density may be expressed as a quadratic function of the classical small strain, the difference between the classical small rotation and the rotation of the micro-structure, and the gradient of the rotation of the micro-structure. Thus, employing the notations

$$A_{j,i} \equiv \frac{\partial A_j}{\partial x_i}, \qquad i = 1, 2, 3,$$

$$A_{[j,i]} \equiv \tfrac{1}{2}(A_{j,i} - A_{i,j}) = -A_{[i,j]},$$

$$B_{[ij]} \equiv \tfrac{1}{2}(B_{ij} - B_{ji}) = -B_{[ji]},$$

we may write, for the potential energy-density,

$$W = W(\varepsilon_{ij}, \gamma_{[ij]}, \kappa_{i[jk]}), \tag{1}$$

where

$$\varepsilon_{ij} \equiv \tfrac{1}{2}(u_{j,i} + u_{i,j}) = \varepsilon_{ji}, \tag{2a}$$

$$\gamma_{[ij]} \equiv u_{[j,i]} - \psi_{[ij]} = -\gamma_{[ji]}, \tag{2b}$$

$$\kappa_{i[jk]} \equiv \psi_{[jk],i} = -\kappa_{i[kj]}, \tag{2c}$$

in which the u_i are the components of displacement and the $\psi_{[ij]}$ are the components of rotation of the micro-structure.

We adopt the definitions

$$\tau_{ij} \equiv \frac{\partial W}{\partial \varepsilon_{ij}} = \tau_{ji}, \tag{3a}$$

$$\sigma_{[ij]} \equiv \frac{\partial W}{\partial \gamma_{[ij]}} = -\sigma_{[ji]}, \tag{3b}$$

$$\mu_{i[jk]} \equiv \frac{\partial W}{\partial \kappa_{i[jk]}} = -\mu_{i[kj]}. \tag{3c}$$

Here τ_{ij} is the classical (Cauchy) stress and $\mu_{i[jk]}$ is the Cosserat couple-stress. The Cosserats considered $\sigma_{[ij]}$ to be the antisymmetric part of an asymmetric tensor whose symmetric part is the Cauchy stress. However, if the Cosserat theory is extended [2] to include strain of the micro-structure, as well as rotation, $\sigma_{[ij]}$ appears naturally as the antisymmetric part of another tensor. This distinction is retained here.

The variation of W is

$$\begin{aligned}\delta W &= \tau_{ij}\delta\varepsilon_{ij} + \sigma_{[ij]}\delta\gamma_{[ij]} + \mu_{i[jk]}\delta\kappa_{i[jk]},\\ &= [(\tau_{ij}+\sigma_{[ij]})\delta u_j]_{,i} - (\tau_{ij}+\sigma_{[ij]})_{,i}\delta u_j - \sigma_{[ij]}\delta\psi_{[ij]} + (\mu_{i[jk]}\delta\psi_{[jk]})_{,i} - \mu_{i[jk],i}\delta\psi_{[jk]}\end{aligned}$$

and, with the divergence theorem,

$$\begin{aligned}\int_V \delta W\,\mathrm{d}V &= \int_S n_i[(\tau_{ij}+\sigma_{[ij]})\delta u_j + \mu_{i[jk]}\delta\psi_{[jk]}]\,\mathrm{d}S\\ &\quad - \int_V [(\tau_{ij}+\sigma_{[ij]})\delta u_j + (\mu_{i[jk],i}+\sigma_{[jk]})\delta\psi_{jk}]\,\mathrm{d}V,\end{aligned} \tag{4}$$

where the n_i are the components of the unit outward normal to the surface S of a volume V.

A principle of virtual work is expressed by

$$\int_V \delta W\,\mathrm{d}V = \int_S (t_j\delta u_j + \mathrm{T}_{[jk]}\delta\psi_{[jk]})\,\mathrm{d}S + \int_V (f_j\delta u_j + \Phi_{[jk]}\delta\psi_{[jk]})\,\mathrm{d}V, \tag{5}$$

where t_j is the surface traction, $\mathrm{T}_{[jk]}$ is the surface couple, f_j is the body force and $\Phi_{[jk]}$ is the body couple.

Upon equating coefficients of like independent variations, δu_i and $\delta\psi_{[jk]}$, in the surface and volume integrals in (4) and (5), we arrive at the Cosserat stress-equations of equilibrium [1]

$$(\tau_{ij}+\sigma_{[ij]})_{,i} + f_j = 0, \tag{6a}$$

$$\mu_{i[jk],i} + \sigma_{[jk]} + \Phi_{[jk]} = 0, \tag{6b}$$

and the boundary conditions

$$t_j = n_i(\tau_{ij}+\sigma_{[ij]}), \tag{7a}$$

$$\mathrm{T}_{[jk]} = n_i\mu_{i[jk]}. \tag{7b}$$

We shall consider only isotropic materials without initial stress. Then the potential energy-density must be a linear function of such product pairs of ε_{ij}, $\gamma_{[ij]}$ and $\kappa_{i[jk]}$ as are scalars. There are two such for ε_{ij} ($\varepsilon_{ii}\varepsilon_{jj}$ and $\varepsilon_{ij}\varepsilon_{ij}$), one for $\gamma_{[ij]}$ ($\gamma_{[ij]}\gamma_{[ij]}$) and three for $\kappa_{i[jk]}$ ($\kappa_{i[ik]}\kappa_{j[kj]}$, $\kappa_{i[jk]}\kappa_{i[jk]}$ and $\kappa_{i[jk]}\kappa_{j[ki]}$) and no others. Hence*

$$W = \tfrac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ij} + \beta\gamma_{[ij]}\gamma_{[ij]} + \alpha_1\kappa_{i[ik]}\kappa_{j[kj]} + \alpha_2\kappa_{i[jk]}\kappa_{i[jk]} + \alpha_3\kappa_{i[jk]}\kappa_{j[ki]}. \tag{8}$$

* The relations between the constants β and α_i in (8) and those in [2] are

$$\beta = \tfrac{1}{2}(b_2-b_3), \qquad \alpha_2 = \tfrac{1}{2}(a_{10}-a_{13}),$$

$$\alpha_1 = a_2 - \tfrac{1}{2}a_3 - \tfrac{1}{2}a_8,$$

$$\alpha_3 = a_{11} - \tfrac{1}{2}a_{14} - \tfrac{1}{2}a_{15}.$$

Then (3) become

$$\tau_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}, \tag{9a}$$

$$\sigma_{[ij]} = 2\beta\gamma_{[ij]}, \tag{9b}$$

$$\mu_{i[jk]} = \alpha_1(\kappa_{l[lj]}\delta_{ik} + \kappa_{l[kl]}\delta_{ij}) + 2\alpha_2\kappa_{i[jk]} + \alpha_3(\kappa_{k[ij]} + \kappa_{j[ki]}). \tag{9c}$$

The forty-two equations (2), (6) and (9) constitute one form of the equations of the linear theory of equilibrium of an isotropic, elastic, Cosserat continuum. As noted previously, these equations have been amplified, in another paper [2], to include a homogeneous strain of the micro-structure. In [2], the complete deformation of the micro-structure is represented by an asymmetric tensor ψ_{ij}. To reduce to the Cosserat continuum, it is only necessary to set the symmetric part of ψ_{ij} equal to zero.

Passage to a limit, as $\beta \to \infty$, $\gamma_{[ij]} \to 0$, reduces the Cosserat continuum to one without micro-structure but with its potential energy-density dependent on strain and gradient of rotation. The resulting equations are those considered in [3].

Upon substituting (2) in (9) and the result in (6), we find the following equations on u_i and $\psi_{[ij]}$:

$$(\lambda+\mu-\beta)u_{j,ji} + (\mu+\beta)u_{i,jj} - 2\beta\psi_{[ji],j} + f_i = 0, \tag{10a}$$

$$(\alpha_1+\alpha_3)(\psi_{[ki],kj} + \psi_{[jk],ki}) + 2\alpha_2\psi_{[ij],kk} - 2\beta\psi_{[ij]} + \beta(u_{j,i} - u_{i,j}) + \Phi_{[ij]} = 0. \tag{10b}$$

STRESS FUNCTIONS

For the purpose of this section, it is convenient to express (10) in an invariant form. Let $\mathbf{u}$ and $\mathbf{f}$ be the vectors with components u_i and f_i; let $\boldsymbol{\psi}^A$, $\boldsymbol{\Phi}^A$ and $\mathbf{I}$ be the dyadics with components $\psi_{[ij]}$, $\Phi_{[ij]}$ and δ_{ij}; and write $\boldsymbol{\Phi}^A = -\frac{1}{2}\mathbf{I}\times\mathbf{c}$, i.e. in terms of a body couple vector $\mathbf{c}$. Then (10) become

$$(\lambda+\mu-\beta)\nabla\nabla\cdot\mathbf{u} + (\mu+\beta)\nabla^2\mathbf{u} - 2\beta\nabla\cdot\boldsymbol{\psi}^A + \mathbf{f} = 0, \tag{11a}$$

$$(\alpha_1+\alpha_3)(\nabla\cdot\boldsymbol{\psi}^A\nabla + \nabla\boldsymbol{\psi}^A\cdot\nabla) + 2\alpha_2\nabla^2\boldsymbol{\psi}^A - 2\beta\boldsymbol{\psi}^A + \beta(\nabla\mathbf{u} - \mathbf{u}\nabla) - \tfrac{1}{2}\mathbf{I}\times\mathbf{c} = 0. \tag{11b}$$

A proof will now be given that any solution ($\mathbf{u}$, $\boldsymbol{\psi}^A$) of (11), in a region V bounded by a surface S, can be expressed as

$$\mathbf{u} = \nabla\times\mathbf{K} + (1 - l_3^2\nabla^2)(\mathbf{B} - l_1^2\nabla\nabla\cdot\mathbf{B}) - \tfrac{1}{2}(k_1 - l_3^2\nabla^2)\nabla[\mathbf{r}\cdot(1 - l_1^2\nabla^2)\mathbf{B} + B_0], \tag{12a}$$

$$\boldsymbol{\psi}^A = -\tfrac{1}{4}\mathbf{I}\times[\nabla^2\nabla(\mathbf{r}\cdot\mathbf{K} + K_0) + 2\nabla\times\mathbf{B}], \tag{12b}$$

where

$$\mu(1 - l_1^2\nabla^2)\nabla^2\mathbf{B} = -\mathbf{f} - \tfrac{1}{2}(1 + \mu/\beta)\nabla\times\mathbf{c}, \tag{13a}$$

$$\mu\nabla^2 B_0 = \mathbf{r}\cdot[\mathbf{f} + \tfrac{1}{2}(1 + \mu/\beta)\nabla\times\mathbf{c}], \tag{13b}$$

$$2\beta\nabla^2\mathbf{K} = \mathbf{c}, \tag{13c}$$

$$2\beta(1 - l_2^2\nabla^2)\nabla^2 K_0 = 4l_2^2\nabla\cdot\mathbf{c} - \mathbf{r}\cdot(1 - l_2^2\nabla^2)\mathbf{c}, \tag{13d}$$

$$k_1 = (\lambda+\mu)/(\lambda+2\mu),$$

$$l_1^2 = (\mu+\beta)(2\alpha_2 - \alpha_1 - \alpha_3)/2\mu\beta,$$

$$l_2^2 = \alpha_2/\beta, \quad l_3^2 = (2\alpha_2 - \alpha_1 - \alpha_3)/\beta,$$

and $\mathbf{r}$ is the position vector.

Consider a field point $P(x, y, z)$ and a source point $Q(\xi, \eta, \zeta)$ and define

$$4\pi \mathbf{U}_P \equiv -\int_V r_1^{-1} \mathbf{u}_Q \, \mathrm{d}V_Q,$$

where $\mathrm{d}V_Q = \mathrm{d}\xi \, \mathrm{d}\eta \, \mathrm{d}\zeta$ and r_1 is the distance between P and Q.

Then $\nabla^2 \mathbf{U} = \mathbf{u}$, or

$$\mathbf{u} = \nabla\nabla \cdot \mathbf{U} - \nabla \times \nabla \times \mathbf{U}. \tag{14}$$

Define

$$\varphi \equiv \nabla \cdot \mathbf{U}, \qquad \mathbf{H} \equiv -\nabla \times \mathbf{U}, \qquad (\nabla \cdot \mathbf{H} = 0).$$

Substitution of these in (14) produces Helmholtz's resolution:

$$\mathbf{u} = \nabla\varphi + \nabla \times \mathbf{H}, \qquad \nabla \cdot \mathbf{H} = 0. \tag{15}$$

Similarly [4] a scalar function χ and a vector function $\mathbf{G}$ can be defined in terms of $\boldsymbol{\psi}^A$ in such a way that

$$\boldsymbol{\psi}^A = -\tfrac{1}{2}\mathbf{I} \times (\nabla\chi - \nabla^2 \mathbf{G}), \qquad \nabla \cdot \mathbf{G} = 0. \tag{16}$$

The substitution of (15) and (16) into (11) converts the latter to*

$$\nabla^2 \{(\lambda + 2\mu)\nabla\varphi + \nabla \times [(\mu + \beta)\mathbf{H} - \beta\mathbf{G}]\} + \mathbf{f} = 0, \tag{17a}$$

$$\beta(1 - l_2^2 \nabla^2)\nabla\chi + \beta\nabla^2[\mathbf{H} - (1 - l_3^2\nabla^2)\mathbf{G}] - \tfrac{1}{2}\mathbf{c} = 0. \tag{17b}$$

From (17b),

$$\beta\nabla^2\nabla \times \mathbf{H} = \beta(1 - l_3^2\nabla^2)\nabla^2\nabla \times \mathbf{G} + \tfrac{1}{2}\nabla \times \mathbf{c}.$$

Upon substituting this expression for $\nabla^2\nabla \times \mathbf{H}$ into (17a), we find

$$\mu\nabla^2[k\nabla\varphi + (1 - l_1^2\nabla^2)\nabla \times \mathbf{G}] = -\mathbf{f} - \tfrac{1}{2}(1 + \mu/\beta)\nabla \times \mathbf{c}, \tag{18}$$

where $k = (\lambda + 2\mu)/\mu$.

Now define

$$4\pi l_1^2 \mathbf{B}'_P \equiv \int_V r_1^{-1} e^{-r_1/l_1} [k\nabla\varphi + (1 - l_1^2\nabla^2)\nabla \times \mathbf{G}]_Q \, \mathrm{d}V_Q.$$

Then

$$(1 - l_1^2\nabla^2)\mathbf{B}' = k\nabla\varphi + (1 - l_1^2\nabla^2)\nabla \times \mathbf{G} \tag{19}$$

and, from (18) and (19),

$$\mu(1 - l_1^2\nabla^2)\nabla^2\mathbf{B}' = -\mathbf{f} - \tfrac{1}{2}(1 + \mu/\beta)\nabla \times \mathbf{c}. \tag{20}$$

Also, from (19),

$$(1 - l_1^2\nabla^2)\nabla \cdot \mathbf{B}' = k\nabla^2\varphi. \tag{21}$$

Define

$$2k\varphi^* \equiv \mathbf{r} \cdot (1 - l_1^2\nabla^2)\mathbf{B}'. \tag{22}$$

* Note that (17) are what (27) of Ref. [4] reduce to when $\boldsymbol{\psi} = \boldsymbol{\psi}^A$.

Then, using (20) and (21), we find

$$2k\mu\nabla^2\varphi^* = 2k\mu\nabla^2\varphi - \mathbf{r}\cdot[\mathbf{f}+\tfrac{1}{2}(1+\mu/\beta)\nabla\times\mathbf{c}]. \tag{23}$$

With the definition $B_0 \equiv 2k(\varphi-\varphi^*)$, we have, from (23) and (22),

$$\mu\nabla^2 B_0 = \mathbf{r}\cdot[\mathbf{f}+\tfrac{1}{2}(1+\mu/\beta)\nabla\times\mathbf{c}], \tag{24}$$

$$2k\varphi = \mathbf{r}\cdot(1-l_1^2\nabla^2)\mathbf{B}'+B_0. \tag{25}$$

Define

$$\mathbf{D} \equiv \mathbf{B}'-l_1^2\nabla\nabla\cdot\mathbf{B}'-k\nabla\varphi.$$

By (21), $\nabla\cdot\mathbf{D} = 0$. Hence there exists a function $\mathbf{G}^*$ such that $\nabla\times\mathbf{G}^* = \mathbf{D}$, or

$$\nabla\times\mathbf{G}^* = \mathbf{B}'-l_1^2\nabla\nabla\cdot\mathbf{B}'-k\nabla\varphi; \tag{26}$$

whence, by (21) and (19),

$$(1-l_1^2\nabla^2)\nabla\times\mathbf{G}^* = (1-l_1^2\nabla^2)\nabla\times\mathbf{G}^*.$$

With the definition $\mathbf{B}'' \equiv \nabla\times\mathbf{G}-\nabla\times\mathbf{G}^*$, we have

$$(1-l_1^2\nabla^2)\mathbf{B}'' = 0, \qquad \nabla\cdot\mathbf{B}'' = 0 \tag{27}$$

and, from (26) and (25),

$$\nabla\times\mathbf{G} = \mathbf{B}''+\mathbf{B}'-l_1^2\nabla\nabla\cdot\mathbf{B}'-\tfrac{1}{2}\nabla[\mathbf{r}\cdot(1-l_1^2\nabla^2)\mathbf{B}'+B_0]. \tag{28}$$

Define $\mathbf{B} \equiv \mathbf{B}'+\mathbf{B}''$. Then, in view of (27), (28) may be written as

$$\nabla\times\mathbf{G} = \mathbf{B}-l_1^2\nabla\nabla\cdot\mathbf{B}-\tfrac{1}{2}\nabla[\mathbf{r}\cdot(1-l_1^2\nabla^2)\mathbf{B}+B_0], \tag{29}$$

while (20) becomes

$$\mu(1-l_1^2\nabla^2)\nabla^2\mathbf{B} = -\mathbf{f}-\tfrac{1}{2}(1+\mu/\beta)\nabla\times\mathbf{c} \tag{30}$$

and (25) becomes

$$2k\varphi = \mathbf{r}\cdot(1-l_1^2\nabla^2)\mathbf{B}+B_0. \tag{31}$$

From (29), $\nabla\times\nabla\times\mathbf{G} = \nabla\times\mathbf{B}$; but $\nabla\times\nabla\times\mathbf{G} = \nabla\nabla\cdot\mathbf{G}-\nabla^2\mathbf{G}$ and, by (16), $\nabla\cdot\mathbf{G} = 0$. Hence

$$\nabla^2\mathbf{G} = -\nabla\times\mathbf{B}. \tag{32}$$

Now define

$$4\pi\chi_P^* \equiv -\int_V r_1^{-1}\chi_Q\,\mathrm{d}V_Q,$$

so that

$$\chi = \nabla^2\chi^*. \tag{33}$$

Substitution of (33) into (17b) yields

$$\beta\nabla^2[(1-l_2^2\nabla^2)\nabla\chi^*+\mathbf{H}-(1-l_3^2\nabla^2)\mathbf{G}]-\tfrac{1}{2}\mathbf{c} = 0. \tag{34}$$

Define

$$\mathbf{K} \equiv (1-l_2^2\nabla^2)\nabla\chi^*+\mathbf{H}-(1-l_3^2\nabla^2)\mathbf{G}.$$

Then

$$2\beta\nabla^2\mathbf{K} = \mathbf{c}, \tag{35}$$

$$\nabla\cdot\mathbf{K} = (1-l_2^2\nabla^2)\nabla^2\chi^*, \tag{36}$$

$$\nabla\times\mathbf{K} = \nabla\times\mathbf{H}-(1-l_3^2\nabla^2)\nabla\times\mathbf{G}. \tag{37}$$

Define

$$\chi^{**} = \tfrac{1}{2}\mathbf{r}\cdot\mathbf{K} \tag{38}$$

and find, using (35) and (36),

$$4\beta(1-l_2^2\nabla^2)\nabla^2(\chi^{**}-\chi^*) = -4l_2^2\nabla\cdot\mathbf{c}+\mathbf{r}\cdot(1-l_2^2\nabla^2)\mathbf{c}. \tag{39}$$

Thus, with the definition

$$K_0 \equiv 2(\chi^*-\chi^{**}), \tag{40}$$

we have, from (39),

$$2\beta(1-l_2^2\nabla^2)\nabla^2K_0 = 4l_2^2\nabla\cdot\mathbf{c}-\mathbf{r}\cdot(1-l_2^2\nabla^2)\mathbf{c}. \tag{41}$$

Finally, from (33), (40) and (38),

$$\chi = \tfrac{1}{2}\nabla^2(\mathbf{r}\cdot\mathbf{K}+K_0). \tag{42}$$

The expression (12a) for $\mathbf{u}$ is given by (15) with φ expressed in terms of $\mathbf{B}$ and B_0 by (31), $\nabla\times\mathbf{H}$ expressed in terms of $\mathbf{K}$ and $\nabla\times\mathbf{G}$ by (37) and $\nabla\times\mathbf{G}$ expressed in terms of $\mathbf{B}$ and B_0 by (29). The expression (12b) for $\boldsymbol{\psi}^A$ is given by (16) with χ expressed in terms of $\mathbf{K}$ and K_0 by (42) and $\nabla^2\mathbf{G}$ expressed in terms of $\nabla\times\mathbf{B}$ by (32). Equations (13) are, respectively, (30), (24), (35) and (41). These equations are elliptic because positive definiteness of the potential energy-density (8) requires

$$\mu > 0, \qquad \beta > 0, \qquad \alpha_2 > 0, \qquad 2\alpha_2-\alpha_1-\alpha_3 > 0,$$

and, hence, l_1^2 and l_2^2 are positive.

CONCENTRATED FORCE AND COUPLE

For the problem of a concentrated force in an infinite region, $\mathbf{c} = 0$ and we may take $\mathbf{K} = 0$, $K_0 = 0$. Then of the four equations (13), there remain only the following on $\mathbf{B}$ and B_0:

$$\mu(1-l_1^2\nabla^2)\nabla^2\mathbf{B} = -\mathbf{f},$$

$$\mu\nabla^2B_0 = \mathbf{r}\cdot\mathbf{f}.$$

These are the same as the equations encountered in [3] except for the replacement of l_1 by another constant. Hence, the solution for a concentrated force $\mathbf{P}$ at the origin in an infinite region has the same form as that found in [3], namely:

$$\mathbf{B} = \frac{\mathbf{P}}{4\pi\mu r}(1-e^{-r/l_1}), \qquad B_0 = 0.$$

If body couples are present, but body forces are absent, the equations (13) on the stress functions are

$$\begin{aligned}2\mu(1-l_1^2\nabla^2)\nabla^2\mathbf{B} &= -(1+\mu/\beta)\nabla\times\mathbf{c},\\ 2\mu\nabla^2 B_0 &= (1+\mu/\beta)\mathbf{r}\cdot\nabla\times\mathbf{c},\\ 2\beta\nabla^2\mathbf{K} &= \mathbf{c},\\ 2\beta(1-l_2^2\nabla^2)\nabla^2 K_0 &= 4l_2^2\nabla\cdot\mathbf{c}-\mathbf{r}\cdot(1-l_2^2\nabla^2)\mathbf{c}.\end{aligned}$$

These have the same forms as equations encountered in the body couple problem considered in [3] and the body force problem considered in [2]. The same methods of integration employed in those papers may be used here for the case of a concentrated couple $\mathbf{C}$ at the origin of an infinite region. After noting that, in [2], the right hand side of (48) should be multiplied by 1/2, we find:

$$\mathbf{B} = -\frac{\mu+\beta}{8\pi\mu\beta}\mathbf{C}\times\nabla\left(\frac{1}{r}-\frac{\exp(-r/l_1)}{r}\right),$$

$$B_0 = 0, \qquad \mathbf{K} = -\frac{\mathbf{C}}{8\pi\beta r},$$

$$K_0 = -\frac{l_2^2}{4\pi\beta}\,\mathbf{C}\cdot\nabla\left(\frac{1}{r}-\frac{\exp(-r/l_2)}{r}\right).$$

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Zusammenfassung—In diesem Beitrag werden die Lineargleichungen eines isotropen, elastischen Cosserat Kontinuums vollständig gelöst, und zwar werden diese in Spannungsfunktionen, welche den Papkovitsch Funktionen der klassischen Elastizitätstheorie analog sind, ausgedrückt. Speziallösungen für eine konzentrierte Kraft und ein Kräftepaar werden ebenfalls angegeben.

Абстракт—В настоящей бумаге выводится полное решение, в терминах функций напряжения, аналогичных функциям Папковича классической упругости, для линейных уравнений изотропного упругого континуума Коссера.

Даются также специальные решения для сосредоточенного усилия и для пары сил.